CHAPITRE 1

Notions de base

1. Calcul littéral

1.1 Rappels de cours

Développer, c'est transformer un produit en une somme.
Factoriser, c'est transformer une somme en un produit.

$\boldsymbol{k(a+b) = ka + kb}$

$\boldsymbol{(a+b)(c+d) = ac + ad + bc + bd}$

$\boldsymbol{(a+b)^2 = a^2 + 2ab + b^2}$

$\boldsymbol{(a-b)^2 = a^2 - 2ab + b^2}$

$\boldsymbol{(a-b)(a+b) = a^2 - b^2}$

$\boldsymbol{k(a+b) = ka + kb}$

$\boldsymbol{-(a+b) = -a - b}$

Exemple 1.

Développer : $-3x(4-5x)$; $(3x+6)^2$; $(5x-1)(1-3x)$

- $-3x(4-5x) = -12x + 15x^2$
- $(3x+6)^2 = 9x^2 + 36x + 36$
- $(5x-1)(1-3x) = 5x - 15x^2 - 1 + 3x = -15x^2 + 8x - 1$

Exemple 2.

Factoriser : $3x^2 + 21x$; $36x^2 - 100$; $25x^2 + 30x + 9$; $16x - 4$

- $3x^2 + 21x = 3 \times x \times x + 7 \times 3 \times x = 3 \times (x+7)$
- $36x^2 - 100 = (6x)^2 - 10^2 = (6x-10)(6x+10)$
- $25x^2 + 30x + 9 = (5x)^2 + 2 \times 5x \times 3 + 3^2 = (5x+3)^2$
- $16x - 4 = 4 \times 4x - 4 \times 1 = 4(4x-1)$

1.2 Exercices

1. Développer, réduire :

$-5(2-x)$; $(2x+1)(x^2-3x+2)$; $(a+b)^3$;

$(7x+2y)(-3)$; $(9x-6)(3+2x)$; $(a-b)^3$;

$3x-(2-5x)$; $(1+x+y)^2$; $(a-b)(a^2+ab+b^2)$.

$(3-4x)^2$; $(11-2x)(11+2x)$;

2. Factoriser :

101^2-99^2 ; $5xy-3x^2y^3$; $4+20x+25x^2$;

$7x-14y+28$; $9x^2-6x+1$; $8xy-32xy^2$.

$2x^2-49$;

1.3 Corrections

1.

$\boldsymbol{-5(2-x)} = -5\times 2+(-5)\times(-x) = -10+5x$

$\boldsymbol{(7x+2y)(-3)} = 7x\times(-3)+2y\times(-3) = -21x-6y$

$\boldsymbol{3x-(2-5x)} = 3x-(+\boldsymbol{2}-\boldsymbol{5x}) = 3x-\boldsymbol{2}+\boldsymbol{5x} = 8x-2$

$\boldsymbol{(3-4x)^2} = 3^2-2\times 3\times 4x+(4x)^2 = 9-24x+16x^2$

$\boldsymbol{(2x+1)(x^2-3x+2)}$
$= 2x\times x^2+2x\times(-3x)+2x\times 2+1\times x^2+1\times(-3x)+1\times 2$
$= 2x^3-6x^2+4x+x^2-3x+2 = 2x^3-5x^2+x+2$

$\boldsymbol{(9x-6)(3+2x)}$
$= 9x\times 3+9x\times 2x-6\times 3-6\times 2x = 18x^2+15x-18$

$\boldsymbol{(1+x+y)^2}$
$= (1+x+y)(1+x+y)$
$= 1+x+y+x+x^2+xy+y+yx+y^2$
$= x^2+y^2+2x+2y+2xy+1$

$\boldsymbol{(11-2x)(11+2x)} = 11^2-(2x)^2 = 121-4x^2$

$\boldsymbol{(a+b)^3} = (a+b)^2\times(a+b) = (a^2+2ab+b^2)(a+b)$
$= a^3+a^2b+2a^2b+2ab^2+b^2a+b^3 = a^3+3a^2b+3ab^2+b^3$

$(\boldsymbol{a}-\boldsymbol{b})^{\boldsymbol{3}} = (a-b)^2 \times (a-b) = (a^2 - 2ab + b^2)(a-b)$
$= a^3 - a^2b - 2a^2b + 2ab^2 + b^2a - b^3$
$= a^3 - 3a^2b + 3ab^2 - b^3$

$(\boldsymbol{a}-\boldsymbol{b})(\boldsymbol{a}^{\boldsymbol{2}} + \boldsymbol{ab} + \boldsymbol{b}^{\boldsymbol{2}}) = a^3 + a^2b + ab^2 - ba^2 - ab^2 - b^3 = a^3 - b^3$

2.

$\boldsymbol{101^2 - 99^2} = (101-99)(101+99) = 2 \times 200 = 400$

$\boldsymbol{7x - 14y} + \boldsymbol{28} = \boldsymbol{7} \times x - 2 \times \boldsymbol{7} \times y + \boldsymbol{7} \times 4 = \boldsymbol{7} \times (x - 2y + 4)$

$\boldsymbol{2x^2 - 49} = \left(\sqrt{2} \times x\right)^2 - 7^2 = \left(x\sqrt{2} - 7\right) \times \left(x\sqrt{2} + 7\right)$
$= \left(\sqrt{2} \times x\right)^2 - 7^2 = \left(x\sqrt{2} - 7\right) \times \left(x\sqrt{2} + 7\right)$

$\boldsymbol{5xy - 3x^2y^3}$
$= \boldsymbol{xy} \times 5 - 3\boldsymbol{x} \times x \times \boldsymbol{y} \times y \times y$
$= \boldsymbol{xy} \times 5 - 3\boldsymbol{xy} \times xy^2 = \boldsymbol{xy} \times (5 - 3xy^2)$

$\boldsymbol{9x^2 - 6x} + \boldsymbol{1} = (3x)^2 - 2 \times 3x \times 1 + 1^2 = (3x-1)^2$

$\boldsymbol{4} + \boldsymbol{20x} + \boldsymbol{25x^2} = (5x+2)^2$

$\boldsymbol{8xy - 32xy^2} = \boldsymbol{8xy} \times 1 - 4 \times \boldsymbol{8xy} \times y = \boldsymbol{8xy} \times (1 - 4y)$

2. Les puissances

2.1 Rappel de cours

$$\boldsymbol{a^n} = \underbrace{\boldsymbol{a} \times \boldsymbol{a} \times \ldots \times \boldsymbol{a}}_{\boldsymbol{n} \text{ facteurs}} \qquad \boldsymbol{a} \in \mathbb{R} \ \boldsymbol{et}\ \boldsymbol{n} \in \mathbb{Z}$$

Règles de calcul	*Exemples*
$\boldsymbol{a^0 = 1}$	$\left(\frac{1}{5}\right)^0 = 1$
$\boldsymbol{a^1 = a}$	$\sqrt{6}^1 = \sqrt{6}$
$\boldsymbol{a^{-1} = \frac{1}{a}}$	$3^{-1} = \frac{1}{3}$
$\boldsymbol{a^{-n} = \frac{1}{a^{-n}}}$	$2^{-5} = \frac{1}{2^5}$
$\boldsymbol{a^n \times a^m = a^{n\times m}}$	$5^3 \times 5^{-9} = 5^{-6}$
$\boldsymbol{\frac{a^n}{a^m} = a^{n-m}}$	$\frac{4^8}{4^{-11}} = 4^{8-(-11)} = 4^{19}$
$\boldsymbol{(a^n)^m = a^{n\times m}}$	$((-6)^2)^5 = (-6)^{10}$
$\boldsymbol{a^n b^n = (a \times b)^n}$	$7^2 \times 3^2 = 21^2$
$\boldsymbol{\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n}$	$\frac{4^6}{3^6} = \left(\frac{4}{3}\right)^6$

2.2 Exercices

1. Calculer :

5^{-3} ; 10^4 ; 7^{-2} ; 6^{-1}

2. Écrire sous forme de puissance :

27 ; 0,125 ; $\frac{4}{25}$; $\frac{8}{216}$

3. Calculer à l'aide des règles sur les puissances :

$3^8 \times 3^{-6}$; $4^3 \times 4^{-1} \times 4^2 \times 4^{-4}$; $\frac{5^8}{5^{-2}}$;

$\frac{7^2 \times 7^{-6}}{7^9}$; $\frac{4^2(4^6)^2}{4^{-5}4^3}$; $\frac{10^2 \times 10^{-3} \times 10^5}{10^{-8} \times 10^6 \times 10^{-7}}$

4. Simplifier :

$\frac{x^3x^{-9}}{x^2x^4}$; $\frac{a^{-8}b^7}{a^2b^5}$; $\left(\frac{a}{b}\right)^3 \times \frac{b^2}{a^{-1}}$

2.3 Corrections

1.

$\mathbf{5^{-3}} = \frac{1}{5^3} = \frac{1}{125}$

$\mathbf{10^4} = 10\,000$

$\mathbf{7^{-2}} = \frac{1}{7^2} = \frac{1}{49}$

$\mathbf{6^{-1}} = \frac{1}{6}$

2.

$\mathbf{27} = 9 \times 3 = 3 \times 3 \times 3 = 3^3$

$\mathbf{0,125} = \frac{1}{8} = \frac{1}{2 \times 2 \times 2} = \frac{1}{2^3}$

$\mathbf{\frac{4}{25}} = \frac{2^2}{5^2} = \left(\frac{2}{5}\right)^2$

$\mathbf{\frac{8}{216}} = \frac{8}{27 \times 8} = \frac{1}{27} = \frac{1}{3^3}$

3.

$\mathbf{3^8 \times 3^{-6}} = 3^{8+(-6)}$
$= 3^2 = 9$

$\mathbf{4^3 \times 4^{-1} \times 4^2 \times 4^{-4}}$
$= 4^{3-1+2-4} = 4^0 = 1$

$\mathbf{\frac{5^8}{5^{-2}}} = 5^{8-(-2)} = 5^{10}$

$\mathbf{\frac{7^2 \times 7^{-6}}{7^9}} = 7^{2+(-6)-9} = 7^{-13}$

$\mathbf{\frac{4^2(4^6)^2}{4^{-5}4^3}} = \frac{4^2 \times 4^{6\times 2}}{4^{-5+3}}$

$= \frac{4^{2+12}}{4^{-2}} = 4^{14-(-2)} = 4^{16}$

$\mathbf{\frac{10^2 \times 10^{-3} \times 10^5}{10^{-8} \times 10^6 \times 10^{-7}}}$

$= 10^{2-3+5+8-6+7} = 10^{13}$

4.

$\boldsymbol{\frac{x^3x^{-9}}{x^2x^4}} = x^{3-9-2-4}$

$= x^{-12}$

$\boldsymbol{\frac{a^{-8}b^7}{a^2b^5}} = a^{-8-2} \times b^{7-5}$

$= a^{-10} \times b^2$

$\boldsymbol{\left(\frac{a}{b}\right)^3 \times \frac{b^2}{a^{-1}}}$

$= \frac{a^3}{b^3} \times \frac{b^2}{a^{-1}}$

$= a^{3+1} \times b^{2-3} = a^4 \times b^{-1}$

3. Les fractions

3.1 Rappel de cours

Exemples

$\boldsymbol{\frac{a}{c}+\frac{b}{c}=\frac{a+b}{c}}$	$\frac{2}{3}-\frac{7}{3}=\frac{2-7}{3}=-\frac{5}{3}$
$\boldsymbol{\frac{a}{c}+\frac{b}{d}=\frac{a\times d+b\times c}{cd}}$	$-\frac{5}{4}+\frac{3}{7}=-\frac{23}{28}$
$\boldsymbol{a\times\frac{b}{c}=\frac{a\times b}{c}}$	$6\times\frac{7}{11}=\frac{6\times 7}{11}=\frac{42}{11}$
$\boldsymbol{\frac{a}{c}\times\frac{b}{d}=\frac{a\times b}{c\times d}}$	$\frac{3}{8}\times\frac{5}{7}=\frac{3\times 5}{8\times 7}=\frac{15}{56}$
$\boldsymbol{\frac{\frac{a}{c}}{\frac{b}{d}}=\frac{a}{c}\times\frac{d}{b}=\frac{a\times d}{c\times b}}$	$\frac{\frac{5}{2}}{\frac{3}{7}}=\frac{5}{2}\times\frac{7}{3}=\frac{35}{6}$
$\boldsymbol{\frac{a\times c}{b\times c}=\frac{a}{b}}$	$\frac{15\times 8}{7\times 8}=\frac{15}{7}$
$\boldsymbol{\frac{-a}{-b}=\frac{a}{b}}$	$\frac{-5}{-2}=\frac{5}{2}$
$\boldsymbol{-\frac{a}{b}=\frac{-a}{b}=\frac{a}{-b}}$	$-\frac{4}{9}=\frac{-4}{9}=\frac{4}{-9}$

3.2 Exercices

1. **Calculer et simplifier les expressions suivantes** :

$\frac{5}{9}+\frac{7}{9}-\frac{4}{9}$; $\frac{5}{13}-\frac{3}{26}$; $4+\frac{2}{7}$; $\frac{1}{5}-\frac{8}{15}+\frac{3}{45}$; $\frac{5}{7}\times\frac{14}{25}$; $\frac{\frac{3}{2}}{\frac{5}{3}}\times\frac{4}{5}$; $\frac{1+\frac{2}{3}}{\frac{3}{4}-\frac{5}{8}}$

$\frac{7x^2yz^6}{14x^4y^3z^2}$; $\frac{x}{4}-\frac{12x}{5}+\frac{3x}{20}$; $\frac{x^2+2x}{x^3}$; $\frac{5y+xy}{y^2}$

2. **Simplifier les fractions après décomposition** :

$\frac{168}{63}$; $\frac{143}{104}$; $\frac{125}{45}$; $\frac{567}{252}$

3. **Simplifier :**

$\frac{2}{x}-\frac{5}{x+3}$; $\frac{x}{x+1}+\frac{2x}{x-1}$; $\frac{\frac{1}{x}-2}{2x+\frac{5}{x+1}}$; $\frac{\frac{1}{x+1}-\frac{1}{x}}{\frac{2}{x}}$; $\frac{\frac{x}{x^2-1}}{\frac{3}{x-1}}$

3.3 Corrections

1.

$$\frac{5}{9}+\frac{7}{9}-\frac{4}{9}=\frac{5+7-4}{9}=\frac{8}{9}$$

$$\frac{5}{13}-\frac{3}{26}=\frac{5\times 2}{13\times 2}-\frac{3}{26}=\frac{7}{26}$$

$$4+\frac{2}{7}=\frac{4\times 7}{1\times 7}+\frac{2}{7}=\frac{30}{7}$$

$$\frac{1}{5}-\frac{8}{15}+\frac{3}{45}=\frac{1\times 9}{5\times 9}-\frac{8\times 3}{15\times 3}+\frac{3}{45}=-\frac{12}{45}=-\frac{4}{15}$$

$$\frac{5}{7}\times\frac{14}{25}=\frac{5\times 14}{7\times 25}=\frac{5\times 2\times 7}{7\times 5\times 5}=\frac{2}{5}$$

$$\frac{\frac{3}{2}}{\frac{5}{3}}\times\frac{4}{5}=\frac{3}{2}\times\frac{3}{5}\times\frac{4}{5}=\frac{36}{50}=\frac{18}{25}$$

$$\frac{1+\frac{2}{3}}{\frac{3}{4}-\frac{5}{8}}=\frac{\frac{5}{3}}{\frac{1}{8}}=\frac{5}{3}\times\frac{8}{1}=\frac{40}{3}$$

$$\frac{7x^2yz^6}{14x^4y^3z^2}=\frac{\cancel{7x^2y}z^4\times\cancel{z^2}}{\cancel{7}\times 2\times\cancel{x^2}\times x^2\times\cancel{y}\times y^2\times\cancel{z^2}}=\frac{z^4}{2x^2y^2}$$

$$\frac{x}{4}-\frac{12x}{5}+\frac{3x}{20}=\frac{5x}{20}-\frac{48x}{20}+\frac{3x}{20}=-\frac{40x}{20}=-2x$$

$$\frac{x^2+2x}{x^3}=\frac{\cancel{x}\times x+2\times\cancel{x}}{\cancel{x}\times x\times x}=\frac{x+2}{x^2}$$

$$\frac{5y+xy}{y^2}=\frac{5\times\cancel{y}+x\times\cancel{y}}{\cancel{y}\times y}=\frac{5+x}{y}$$

2.

$$\frac{168}{63}=\frac{8\times 7\times 3}{3\times 3\times 7}=\frac{8}{3}$$

$$\frac{125}{45}=\frac{25\times 5}{9\times 5}=\frac{25}{9}$$

$$\frac{143}{104}=\frac{11\times 13}{8\times 13}=\frac{11}{8}$$

$$\frac{567}{252}=\frac{9\times 63}{9\times 28}=\frac{63}{28}=\frac{9\times 7}{4\times 7}=\frac{9}{4}$$

3.

$$\frac{2}{x}-\frac{5}{x+3}=\frac{2(x+3)}{x(x+3)}-\frac{5x}{x(x+3)}=\frac{2x+6-5x}{x(x+3)}=\frac{-3x+6}{x(x+3)}$$

$$\frac{x}{x+1}+\frac{2x}{x-1}=\frac{x(x-1)}{(x+1)(x-1)}+\frac{2x(x+1)}{(x-1)(x+1)}=\frac{x^2-x+2x^2+2x}{x^2-1}=\frac{3x^2+x}{x^2-1}$$

$$\frac{\frac{1}{x}-2}{2x+\frac{5}{x+1}} = \frac{\frac{1-2x}{x}}{\frac{2x(x+1)+5}{x+1}} = \frac{1-2x}{x} \times \frac{x+1}{2x^2+2x+5} = \frac{(1-2x)(x+1)}{x(2x^2+2x+5)}$$

$$\frac{\frac{1}{x+1}-\frac{1}{x}}{\frac{2}{x}} = \frac{\frac{x-(x+1)}{x(x+1)}}{\frac{2}{x}} = \frac{x-(x+1)}{x(x+1)} \times \frac{x}{2} = \frac{(x-x-1)\times x}{2x(x+1)} = -\frac{x}{2x(x+1)}$$

$$\frac{\frac{x}{x^2-1}}{\frac{3}{x-1}} = \frac{x}{x^2-1} \times \frac{x-1}{3} = \frac{x(x-1)}{3(x-1)(x+1)} = \frac{x}{3(x+1)}$$

4. Racines

4.1 Rappel de cours

La racine carrée d'un nombre positif a est l'unique nombre, noté $\sqrt{a}$, tel que $\left(\sqrt{a}\right)^2 = a = a^{\frac{1}{2}}$.

$$\sqrt{a} \times \sqrt{b} = \sqrt{a \times b} \; ; \; \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

Remarque : $\sqrt{a} + \sqrt{b} \neq \sqrt{a+b}$

Ex: $\sqrt{4} + \sqrt{9} = 2 + 3 = 5 \neq \sqrt{4+9} = \sqrt{13}$

Résolution de $x^2 = a$:
Si $a < 0$, il n'y a pas de solution.
Si $a = 0$, il y a une solution, $x = 0$.
Si $a > 0$, il y a deux solutions : $\sqrt{a}$ ou $-\sqrt{a}$.

Exemple 1.

$x^2 = 144 \Leftrightarrow x = \pm 12$

On appelle racine n^ième^ d'un nombre a positif le nombre positif noté $a^{\frac{1}{n}}$ tel que $\left(a^{\frac{1}{n}}\right)^n = a \; ; n \neq 0$

Exemple 2.

$\sqrt[3]{64} = 4 \; car \; 4^3 = 64$;

Notation : $a^{\frac{1}{n}} = \sqrt[n]{a}$